Exact order of discrepancy of normal numbers

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Online Seminar in Diophantine Approximation and Related Topics
Tuesday, February 28, 14:00 CEST

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In the talk we consider the still open question of Korobov: What is the best possible order of discrepancy D_N in N, a sequence $(\{2^n\alpha\})_{n\geq 0}$, can have for some real number α? If $\lim_{N\to\infty} D_N = 0$ then α in called normal in base 2.

So far the best upper bound for D_N for an explicitly known normal number in base 2 is of the form $ND_N \ll \log^2 N$. The first such explicit example is due to Levin (1999), that was later generalized by Becher and Carton (2019). In this talk we discuss the recent result in joint work with Gerhard Larcher that guarantees $ND_N \gg \log^2 N$ for Levin's binary normal number. So EITHER $ND_N \ll \log^2 N$ is the best possible order for D_N in N of a normal number OR there exist another example of a binary normal number with a better growth of ND_N in N. The recent result for Levin's normal number might support the conjecture that $ND_N \ll \log^2 N$ is the best order for D_N in N a normal number can obtain.